

Neutrinos in cosmology

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Abstract

I review the basics of neutrino cosmology, from the question of neutrino decoupling and the presence of sterile neutrinos to the effects of neutrinos on the cosmic microwave background and large scale structure. Particular emphasis is put on cosmological neutrino mass measurements, both the present bounds and the future prospects.

1 Introduction

Neutrinos are the second most abundant particles in our Universe. This means they have a profound impact on many different aspects of cosmology, from the question of leptogenesis in the very early universe, over big bang nucleosynthesis, to late time structure formation. Here I review some general aspects of neutrino cosmology with particular emphasis on issues relevant to cosmological bounds on the neutrino mass.

The absolute value of neutrino masses are very difficult to measure experimentally. On the other hand, mass differences between the light neutrino mass eigenstates, (m_1, m_2, m_3) , can be measured in neutrino oscillation experiments.

The combination of all currently available data suggests two important mass differences in the neutrino mass hierarchy. The solar mass difference of $\delta m_{12}^2 \simeq 7.1 - 8.9 \times 10^{-5} \text{ eV}^2$ (3σ) and the atmospheric mass difference $\delta m_{23}^2 \simeq 1.4 - 3.3 \times 10^{-3} \text{ eV}^2$ (3σ) [1, 4, 2, 3] (see [5] for a recent review).

In the simplest case where neutrino masses are hierarchical these results suggest that $m_1 \sim 0$, $m_2 \sim \delta m_{\text{solar}}$, and $m_3 \sim \delta m_{\text{atmospheric}}$. If the hierarchy is inverted one instead finds $m_3 \sim 0$, $m_2 \sim \delta m_{\text{atmospheric}}$, and $m_1 \sim \delta m_{\text{atmospheric}}$. However, it is also possible that neutrino masses are degenerate, $m_1 \sim m_2 \sim m_3 \gg \delta m_{\text{atmospheric}}$, in which case oscillation experiments are not useful for determining the absolute mass scale.

Experiments which rely on kinematical effects of the neutrino mass offer the strongest probe of this overall mass scale. Tritium decay measurements have been able to put an upper limit on the electron neutrino mass of 2.3 eV (95% conf.) [6]. However, cosmology at present yields an much stronger limit which is also based on the kinematics of neutrino mass.

Very interestingly there is also a claim of direct detection of neutrinoless double beta decay in the Heidelberg-Moscow experiment [7, 8], corresponding to an effective neutrino mass in the 0.1 – 0.9 eV range. If this result is confirmed then it shows that neutrino masses are almost degenerate and well within reach of cosmological detection in the near future.

Another important question which can be answered by cosmological observations is how large the total neutrino energy density is. Apart from the standard model prediction of three light neutrinos, such energy density can be either in the form of additional, sterile neutrino degrees of freedom, or a non-zero neutrino chemical potential.

In section 2 I review the present cosmological data which can be used for analysis of neutrino physics. In section 3 I discuss neutrino physics around the epoch of neutrino decoupling at a temperature of roughly 1 MeV, including the relation between neutrinos and Big Bang nucleosynthesis. Section 4 discusses neutrinos as dark matter particles, including mass constraints on light neutrinos, and sterile neutrino dark matter. Finally, section 5 contains a discussion.

2 Cosmological data

Large Scale Structure (LSS) – At present there are two large galaxy surveys of comparable size, the Sloan Digital Sky Survey (SDSS) [9, 10] and the 2dFGRS (2 degree Field Galaxy Redshift Survey) [11]. The SDSS is still ongoing, but will be completed very soon. It will then be significantly larger than the 2dF. Furthermore, a continuation in the form of a new and extended survey, SDSS-II, has already been approved (see <http://www.sdss.org>).

Cosmic Microwave Background (CMB) – The CMB temperature fluctuations are conveniently described in terms of the spherical harmonics power spectrum $C_l^{TT} \equiv \langle |a_{lm}|^2 \rangle$, where $\frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$. Since Thomson scattering polarizes light, there are also power spectra coming from the polarization. The polarization can be divided into a curl-free ((E)) and a curl ((B)) component, much in the same way as \vec{E} and \vec{B} in electrodynamics can be derived from the gradient of a scalar field and the curl of a vector field respectively (see for instance [12] for a very detailed treatment). The polarization introduced a sequence of new power spectra, but because of different parity some of them are explicitly zero. Altogether there are four independent power spectra: C_l^{TT} , C_l^{EE} , C_l^{BB} , and the T - E cross-correlation C_l^{TE} .

The WMAP experiment has reported data only on C_l^{TT} and C_l^{TE} as described in Refs. [13, 14]. Other experiments, while less precise in the measurement of the temperature anisotropy and not providing full-sky coverage, are much more sensitive to small scale anisotropies and to CMB polarization. Particularly the ground based CBI [16], DASI [17], and ACBAR [15] experiments, as well as the BOOMERANG balloon experiment [18, 19, 20] have provided useful data.

Type Ia supernovae Observations of distant supernovae have been carried out on a large scale for about a decade. In 1998 two different projects almost simultaneously published measurements of about 50 distant type Ia supernovae, out to a redshift of about 0.8 [21, 22]. These measurements were instrumental for the measurement of the late time expansion rate of the universe.

Since then a, new supernovae have continuously been added to the sample, with the Riess et al. [23] "gold" data set of 157 distant supernovae being the most recent. This includes several supernovae measured by the Hubble Space Telescope out to a redshift of 1.7.

Very recently, the first data has been released from the Supernova Legacy Survey (SNLS) [24], providing the currently largest data set of Type Ia supernovae.

Other data – Apart from CMB, LSS and SNI-a data there are a number of other cosmological measurements of importance to neutrino cosmology. Perhaps the most important is the measurement of the Hubble constant by the HST Hubble Key Project, $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [25].

3 Neutrino Decoupling

3.1 Standard model

In the standard model neutrinos interact via weak interactions with e^+ and e^- . In the absence of oscillations neutrino decoupling can be followed via the Boltzmann equation for the single particle distribution function [27]

$$\frac{\partial f}{\partial t} - Hp \frac{\partial f}{\partial p} = C_{\text{coll}}, \quad (1)$$

where C_{coll} represents all elastic and inelastic interactions. In the standard model all these interactions are $2 \leftrightarrow 2$ interactions in which case the collision integral for process i can be written

$$C_{\text{coll},i}(f_1) = \frac{1}{2E_1} \int \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} \frac{d^3\mathbf{p}_3}{2E_3(2\pi)^3} \frac{d^3\mathbf{p}_4}{2E_4(2\pi)^3} \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 + p_4) \Lambda(f_1, f_2, f_3, f_4) S |M|_{12 \rightarrow 34,i}^2, \quad (2)$$

where $S|M|_{12 \rightarrow 34,i}^2$ is the spin-summed and averaged matrix element including the symmetry factor $S = 1/2$ if there are identical particles in initial or final states. The phase-space factor is $\Lambda(f_1, f_2, f_3, f_4) = f_3 f_4 (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_3)(1 - f_4)$.

The matrix elements for all relevant processes can for instance be found in Ref. [28]. If Maxwell-Boltzmann statistics is used for all particles, and neutrinos are assumed to be in complete scattering equilibrium so that they can be represented by a single temperature, then the collision integral can be integrated to yield the average annihilation rate for a neutrino

$$\Gamma = \frac{16G_F^2}{\pi^3} (g_L^2 + g_R^2) T^5, \quad (3)$$

where

$$g_L^2 + g_R^2 = \begin{cases} \sin^4 \theta_W + (\frac{1}{2} + \sin^2 \theta_W)^2 & \text{for } \nu_e \\ \sin^4 \theta_W + (-\frac{1}{2} + \sin^2 \theta_W)^2 & \text{for } \nu_{\mu,\tau} \end{cases}. \quad (4)$$

This rate can then be compared with the Hubble expansion rate

$$H = 1.66 g_*^{1/2} \frac{T^2}{M_{\text{Pl}}} \quad (5)$$

to find the decoupling temperature from the criterion $H = \Gamma|_{T=T_D}$. From this one finds that $T_D(\nu_e) \simeq 2.4$ MeV, $T_D(\nu_{\mu,\tau}) \simeq 3.7$ MeV, when $g_* = 10.75$, as is the case in the standard model.

This means that neutrinos decouple at a temperature which is significantly higher than the electron mass. When e^+e^- annihilation occurs around $T \sim m_e/3$, the neutrino temperature is unaffected whereas the photon temperature is heated by a factor $(11/4)^{1/3}$. The relation $T_\nu/T_\gamma = (4/11)^{1/3} \simeq 0.71$ holds to a precision of roughly one percent. The main correction comes from a slight heating of neutrinos by e^+e^- annihilation, as well as finite temperature QED effects on the photon propagator [29, 30, 31, 32, 33, 34, 28, 35, 36, 37, 38, 39, 40, 41, 44].

3.2 Big Bang nucleosynthesis and the number of neutrino species

Shortly after neutrino decoupling the weak interactions which keep neutrons and protons in statistical equilibrium freeze out. Again the criterion $H = \Gamma|_{T=T_{\text{freeze}}}$ can be applied to find that $T_{\text{freeze}} \simeq 0.5 g_*^{1/6}$ MeV [27].

Eventually, at a temperature of roughly 0.2 MeV deuterium starts to form, and very quickly all free neutrons are processed into ${}^4\text{He}$. The final helium abundance is therefore roughly given by

$$Y_P \simeq \frac{2n_n/n_p}{1 + n_n/n_p} \Big|_{T \simeq 0.2 \text{ MeV}}. \quad (6)$$

n_n/n_p is determined by its value at freeze out, roughly by the condition that $n_n/n_p|_{T=T_{\text{freeze}}} \sim e^{-(m_n-m_p)/T_{\text{freeze}}}$.

Since the freeze-out temperature is determined by g_* this in turn means that g_* can be inferred from a measurement of the helium abundance. However, since Y_P is a function of both $\Omega_b h^2$ and g_* it is necessary to use other measurements to constrain $\Omega_b h^2$ in order to find a bound on g_* . One customary method for doing this has been to use measurements of primordial deuterium to infer $\Omega_b h^2$ and from that calculate a bound on g_* . Usually such bounds are expressed in terms of the equivalent number of neutrino species, $N_\nu \equiv \rho/\rho_{\nu_0}$, instead of g_* . The exact value of the bound is quite uncertain because there are different and inconsistent measurements of the primordial helium abundance (see for instance Ref. [42] for a discussion of this issue). The most recent analyses are [42] where a value of $1.7 \leq N_\nu \leq 3.0$ (95% C.L.) was found, [43] which found $-1.14 \leq \Delta N_\nu \leq 0.73$, and [45] which found that $N_\nu = 3.14^{+0.7}_{-0.65}$ at 68% C.L. The difference in these results can be attributed to different assumptions about uncertainties in the primordial helium abundance.

Another interesting parameter which can be constrained by the same argument is the neutrino chemical potential, $\xi_\nu = \mu_\nu/T$ [47, 48, 49, 50]. At first sight this looks like it is completely equivalent to constraining N_ν . However, this is not true because a chemical potential for electron neutrinos directly influences the $n-p$ conversion rate. Therefore the bound on ξ_{ν_e} from BBN alone is relatively stringent ($-0.1 \leq \xi_{\nu_e} \leq 1$ [47]) compared to that for muon and tau neutrinos ($|\xi_{\nu_{\mu,\tau}}| < 7$ [47]). However, as will be seen in the next section, neutrino oscillations have the effect of almost equilibrating the neutrino chemical potentials prior to BBN, completely changing this conclusion.

3.3 The number of neutrino species - joint CMB and BBN analysis

The BBN bound on the number of neutrino species presented in the previous section can be complemented by a similar bound from observations of the CMB and large scale structure. The CMB depends on N_ν mainly because of the early Integrated Sachs Wolfe effect which increases fluctuation power at scales slightly larger than the first acoustic peak. The large scale structure spectrum depends on N_ν because the scale of matter-radiation equality is changed by varying N_ν .

Many recent papers have used CMB, LSS, and SNI-a data to calculate bounds on N_ν [52, 51, 53, 42, 46], and some of the bounds are listed in Table 3.3. Recent analyses combining BBN, CMB, and large scale structure data can be found in [51, 42], and these results are also listed in Table 3.3.

Common for all the bounds is that $N_\nu = 0$ is ruled out by both BBN and CMB/LSS. This has the important consequence that the cosmological neutrino background has been positively detected, not only during the BBN epoch, but also much later, during structure formation.

The most recent bound which uses the SNI-a "gold" data set, as well as the new Boomerang CMB data finds a limit of $N_\nu = 4.2^{+1.7}_{-1.2}$ at 95% C.L. The bound from late-time observations is now as good as that from BBN, and the two derived value are mutually consistent given the systematic uncertainties in the primordial helium value.

In Fig. 1 we show the currently allowed region for $\Omega_b h^2$ and N_ν (taken from [54]).

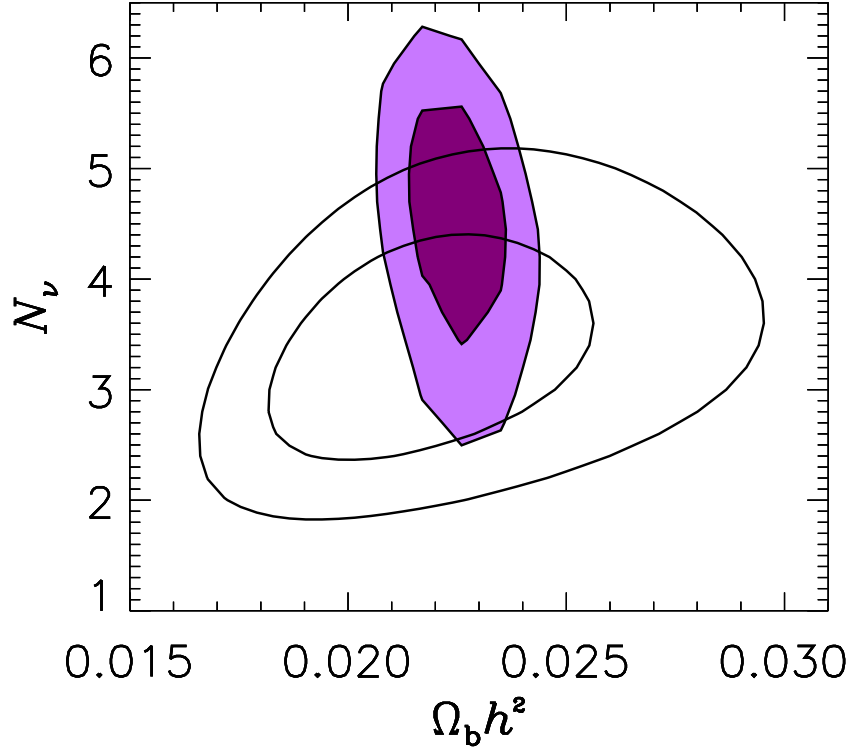


Figure 1: The 68% (dark) and 95% (light) likelihood contours for $\Omega_b h^2$ and N_ν for all available data. The other contours are 68% and 95% regions for BBN, assuming the ^4He and D values given in [45].

Table 1: Various recent limits on the effective number of neutrino species, as well as the data used.

Ref.	Bound on N_ν	Data used
Crotty et al. [52]	$1.4 \leq N_\nu \leq 6.8$	CMB, LSS
Hannestad [51]	$0.9 \leq N_\nu \leq 7.0$	CMB, LSS
Pierpaoli [53]	$1.9 \leq N_\nu \leq 6.62$	CMB, LSS
Barger et al. [42]	$0.9 \leq N_\nu \leq 8.3$	CMB
Hannestad [54]	$3.0 \leq N_\nu \leq 5.9$	CMB, LSS, SNI-a

3.4 The effect of oscillations

In the previous section the one-particle distribution function, f , was used to describe neutrino evolution. However, for neutrinos the mass eigenstates are not equivalent to the flavour eigenstates because neutrinos are mixed. Therefore the evolution of the neutrino ensemble is not in general described by the three scalar functions, f_i , but rather by the evolution of the neutrino density matrix, $\rho \equiv \psi\psi^\dagger$, the diagonal elements of which correspond to f_i .

For three-neutrino oscillations the formalism is quite complicated. However, the difference in Δm_{12} and Δm_{23} , as well as the fact that $\sin 2\theta_{13} \ll 1$ means that the problem effectively reduces to a 2×2 oscillation problem in the standard model. A detailed account of the physics of neutrino oscillations in the early universe is outside the scope of the present paper, however an excellent and very thorough review can be found in Ref. [55]

Without oscillations it is possible to compensate a very large chemical potential for muon and/or tau neutrinos with a small, negative electron neutrino chemical potential [47]. However, since neutrinos are almost maximally mixed a chemical potential in one flavour can be shared with other flavours, and the end result is that during BBN all three flavours have almost equal chemical potential. This in turn means that the bound on ν_e applies to all species so that [56, 57, 58, 59, 60].

$$|\xi_i| = \frac{|\eta_i|}{T} < 0.15 \quad (7)$$

for $i = e, \mu, \tau$.

In models where sterile neutrinos are present even more remarkable oscillation phenomena can occur. However, I do not discuss this possibility further, except for the possibility of sterile neutrino warm dark matter, and instead refer to the review [55].

3.5 Low reheating temperature and neutrinos

In most models of inflation the universe enters the normal, radiation dominated epoch at a reheating temperature, T_{RH} , which is of order the electroweak scale or higher. However, in principle it is possible that this reheating temperature is much lower, of order MeV. This possibility has been studied many times in the literature, and a very general bound of $T_{\text{RH}} \geq 1$ MeV has been found [61, 62, 63, 64]

This very conservative bound comes from the fact that the light element abundances produced by big bang nucleosynthesis disagree with observations if the universe is matter dominated during BBN. However, a somewhat more stringent bound can be obtained by looking at neutrino thermalization during reheating. If a scalar particle is responsible for reheating then direct decay to neutrinos is suppressed because of the necessary helicity flip. This means that if the reheating temperature is too low neutrinos never thermalize. If this is the case then BBN predicts the wrong light element abundances. However, even if the heavy particle has a significant branching ratio into neutrinos there are problems with BBN. The reason is that neutrinos produced in decays are born with energies which are much higher than thermal. If the reheating temperature is too low then a population of high energy neutrinos will remain and also lead to conflict with observed light element abundances. A recent analysis showed that in general the reheating temperature cannot be below roughly 4 MeV [65].

4 Neutrino Dark Matter

Neutrinos are a source of dark matter in the present day universe simply because they contribute to Ω_m . The present temperature of massless standard model neutrinos is $T_{\nu,0} = 1.95 K = 1.7 \times 10^{-4}$ eV, and any neutrino with $m \gg T_{\nu,0}$ behaves like a standard non-relativistic dark matter particle.

The present contribution to the matter density of N_ν neutrino species with standard weak interactions is given by

$$\Omega_\nu h^2 = N_\nu \frac{m_\nu}{92.5 \text{ eV}} \quad (8)$$

Just from demanding that $\Omega_\nu \leq 1$ one finds the bound [66, 67]

$$m_\nu < \frac{46 \text{ eV}}{N_\nu} \quad (9)$$

4.1 The Tremaine-Gunn bound

If neutrinos are the main source of dark matter, then they must also make up most of the galactic dark matter. However, neutrinos can only cluster in galaxies via energy loss due to gravitational relaxation since they do not suffer inelastic collisions. In distribution function language this corresponds to phase mixing of the distribution function [68]. By using the theorem that the phase-mixed or coarse grained distribution function must explicitly take values smaller than the maximum of the original distribution function one arrives at the condition

$$f_{\text{CG}} \leq f_{\nu, \text{max}} = \frac{1}{2} \quad (10)$$

Because of this upper bound it is impossible to squeeze neutrino dark matter beyond a certain limit [68]. For the Milky Way this means that the neutrino mass must be larger than roughly 25 eV *if* neutrinos make up the dark matter. For irregular dwarf galaxies this limit increases to 100-300 eV [69, 70], and means that standard model neutrinos cannot make up a dominant fraction of the dark matter. This bound is generally known as the Tremaine-Gunn bound.

Note that this phase space argument is a purely classical argument, it is not related to the Pauli blocking principle for fermions (although, by using the Pauli principle $f_\nu \leq 1$ one would arrive at a similar, but slightly weaker limit for neutrinos). In fact the Tremaine-Gunn bound works even for bosons if applied in a statistical sense [69], because even though there is no upper bound on the fine grained distribution function, only a very small number of particles reside at low momenta (unless there is a condensate). Therefore, although the exact value of the limit is model dependent, limit applies to any species that was once in thermal equilibrium. A notable counterexample is non-thermal axion dark matter which is produced directly into a condensate.

4.2 Neutrino hot dark matter

A much stronger upper bound on the neutrino mass than the one in Eq. (9) can be derived by noticing that the thermal history of neutrinos is very different from that of a WIMP because the neutrino only becomes non-relativistic very late.

In an inhomogeneous universe the Boltzmann equation for a collisionless species is [71]

$$L[f] = \frac{Df}{D\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau} \frac{\partial f}{\partial x^i} + \frac{dq^i}{d\tau} \frac{\partial f}{\partial q^i} = 0, \quad (11)$$

where τ is conformal time, $d\tau = dt/a$, and $q^i = ap^i$ is comoving momentum. The second term on the right-hand side has to do with the velocity of the distribution in a given spatial point and the third term is the cosmological momentum redshift.

Following Ma and Bertschinger [71] this can be rewritten as an equation for Ψ , the perturbed part of f

$$f(x^i, q^i, \tau) = f_0(q) \left[1 + \Psi(x^i, q^i, \tau) \right] \quad (12)$$

In synchronous gauge that equation is

$$\frac{1}{f_0}[f] = \frac{\partial \Psi}{\partial \tau} + i \frac{q}{\epsilon} \mu \Psi + \frac{d \ln f_0}{d \ln q} \left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2} \mu^2 \right] = \frac{1}{f_0} C[f], \quad (13)$$

where $q^j = q n^j$, $\mu \equiv n^j \hat{k}_j$, and $\epsilon = (q^2 + a^2 m^2)^{1/2}$. k^j is the comoving wavevector. h and η are the metric perturbations, defined from the perturbed space-time metric in synchronous gauge [71]

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \quad (14)$$

$$h_{ij} = \int d^3 k e^{i\vec{k} \cdot \vec{x}} \left(\hat{k}_i \hat{k}_j h(\vec{k}, \tau) + (\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij}) 6\eta(\vec{k}, \tau) \right). \quad (15)$$

Expanding this in Legendre polynomials one arrives at a set of hierarchy equations

$$\begin{aligned} \dot{\delta} &= -\frac{4}{3}\theta - \frac{2}{3}\dot{h} \\ \dot{\theta} &= k^2 \left(\frac{\delta}{4} - \sigma \right) \\ 2\dot{\sigma} &= \frac{8}{15}\theta - \frac{3}{15}kF_3 + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta} \\ \dot{F}_l &= \frac{k}{2l+1} (lF_{l-1} - (l+1)F_{l+1}) \end{aligned} \quad (16)$$

For subhorizon scales ($\dot{h} = \dot{\eta} = 0$) this reduces to the form

$$\begin{aligned} \dot{\delta} &= -\frac{4}{3}\theta \\ \dot{\theta} &= k^2 \left(\frac{\delta}{4} - \sigma \right) \\ 2\dot{\sigma} &= \frac{8}{15}\theta - \frac{3}{15}kF_3 \\ \dot{F}_l &= \frac{k}{2l+1} (lF_{l-1} - (l+1)F_{l+1}) \end{aligned} \quad (17)$$

One should notice the similarity between this set of equations and the evolution hierarchy for spherical Bessel functions. Indeed the exact solution to the hierarchy is

$$F_l(k\tau) \sim j_l(k\tau) \quad (18)$$

This shows that the solution for δ is an exponentially damped oscillation. On small scales, $k > \tau$, perturbations are erased.

This is intuitively understandable in terms of free-streaming. Indeed the Bessel function solution comes from the fact that neutrinos are considered massless. In the limit of CDM the evolution hierarchy is truncated by the fact that $\theta = 0$, so that the CDM perturbation equation is simply $\dot{\delta} = -\dot{h}/2$. For massless particles the free-streaming length is $\lambda = c\tau$ which is reflected in the solution to the Boltzmann hierarchy. Of course the solution only applies when neutrinos are strictly massless. Once $T \sim m$ there is a smooth transition to the CDM solution. Therefore the final solution can be separated into two parts: 1) $k > \tau(T = m)$: Neutrino perturbations are exponentially damped 2) $k < \tau(T = m)$: Neutrino perturbations follow the CDM perturbations. Calculating the free streaming wavenumber in a flat CDM cosmology leads to the simple numerical relation (applicable only for $T_{\text{eq}} \gg m \gg T_0$) [27]

$$\lambda_{\text{FS}} \sim \frac{20 \text{ Mpc}}{\Omega_x h^2} \left(\frac{T_x}{T_\nu} \right)^4 \left[1 + \log \left(3.9 \frac{\Omega_x h^2}{\Omega_m h^2} \left(\frac{T_\nu}{T_x} \right)^2 \right) \right]. \quad (19)$$

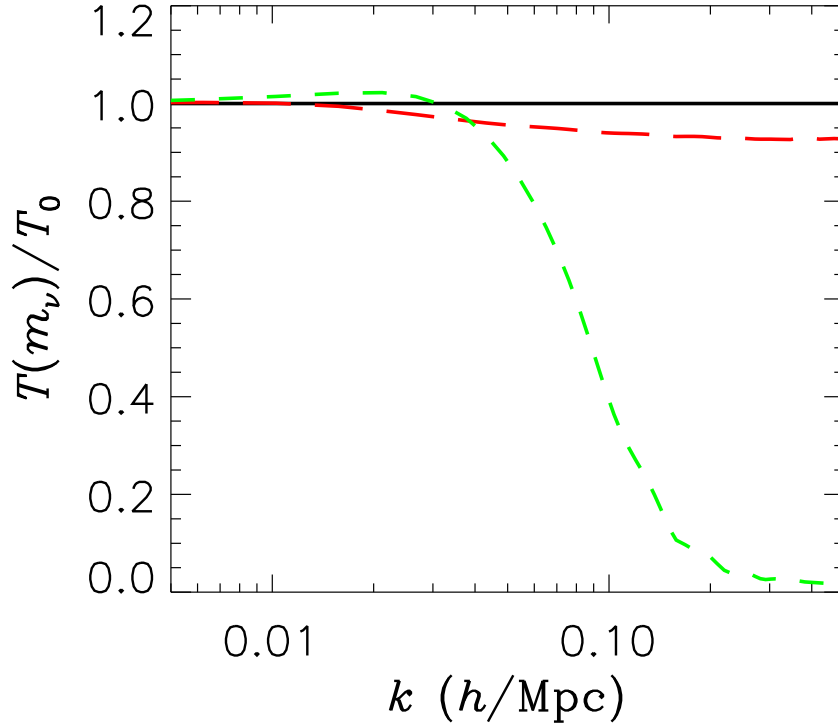


Figure 2: The transfer function $T(k, t = t_0)$ for various different neutrino masses. The solid (black) line is for $m_\nu = 0$, the long-dashed for $m_\nu = 0.3$ eV, and the dashed for $m_\nu = 1$ eV.

In Fig. 2 I have plotted transfer functions for various different neutrino masses in a flat Λ CDM universe ($\Omega_m + \Omega_\nu + \Omega_\Lambda = 1$). The parameters used were $\Omega_b = 0.04$, $\Omega_{\text{CDM}} = 0.26 - \Omega_\nu$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $n = 1$.

When measuring fluctuations it is customary to use the power spectrum, $P(k, \tau)$, defined as

$$P(k, \tau) = |\delta|^2(\tau). \quad (20)$$

The power spectrum can be decomposed into a primordial part, $P_0(k)$, and a transfer function $T(k, \tau)$,

$$P(k, \tau) = P_0(k)T(k, \tau). \quad (21)$$

The transfer function at a particular time is found by solving the Boltzmann equation for $\delta(\tau)$.

At scales much smaller than the free-streaming scale the present matter power spectrum is suppressed roughly by the factor [72]

$$\frac{\Delta P(k)}{P(k)} = \frac{\Delta T(k, \tau = \tau_0)}{T(k, \tau = \tau_0)} \simeq -8 \frac{\Omega_\nu}{\Omega_m}, \quad (22)$$

as long as $\Omega_\nu \ll \Omega_m$. The numerical factor 8 is derived from a numerical solution of the Boltzmann equation, but the general structure of the equation is simple to understand. At scales smaller than the free-streaming scale the neutrino perturbations are washed out completely, leaving only perturbations in the non-relativistic matter (CDM and baryons). Therefore the *relative* suppression of power is proportional to the ratio of neutrino energy density to the overall matter density. Clearly the above relation only applies when $\Omega_\nu \ll \Omega_m$, when Ω_ν becomes dominant the spectrum suppression becomes exponential as in the pure hot dark matter model. This effect is shown for different neutrino masses in Fig. 2.

The effect of massive neutrinos on structure formation only applies to the scales below the free-streaming length. For neutrinos with masses of several eV the free-streaming scale is smaller than the

scales which can be probed using present CMB data and therefore the power spectrum suppression can be seen only in large scale structure data. On the other hand, neutrinos of sub-eV mass behave almost like a relativistic neutrino species for CMB considerations. The main effect of a small neutrino mass on the CMB is that it leads to an enhanced early ISW effect. The reason is that the ratio of radiation to matter at recombination becomes larger because a sub-eV neutrino is still relativistic or semi-relativistic at recombination. With the WMAP data alone it is very difficult to constrain the neutrino mass, and to achieve a constraint which is competitive with current experimental bounds it is necessary to include LSS data from 2dF or SDSS. When this is done the bound becomes very strong, somewhere in the range of 1-1.5 eV for the sum of neutrino masses, depending on assumptions about priors [14, 51, 82, 10, 87, 75, 76, 77, 78, 79, 80]

The bound can be strengthened even further by including data from the Lyman- α forest, measured at large redshifts. This was done for instance in [76, 77], where bounds as strong as 0.4 eV were derived. However, the systematic errors in extracting the matter power spectrum from the Lyman- α flux power spectrum are at present not understood at a level where this bound can be claimed to be robust.

4.2.1 Parameter degeneracy with the dark energy equation of state

One caveat of most cosmological parameter analyses used to probe particle physics is that they are done with the minimal cosmological standard model. In principle there could easily be additional parameters which are important. One such example was described in [80], where it was shown that there is an almost perfect degeneracy between the neutrino mass and the equation of state of the dark energy, w . This degeneracy is shown in Fig. 3: An increasing $\sum m_\nu$ can be compensated by decreasing w . While for low neutrino masses a cosmological constant ($w = -1$) is allowed, for high neutrino masses only dark energy models in the phantom regime ($w < -1$) are allowed.

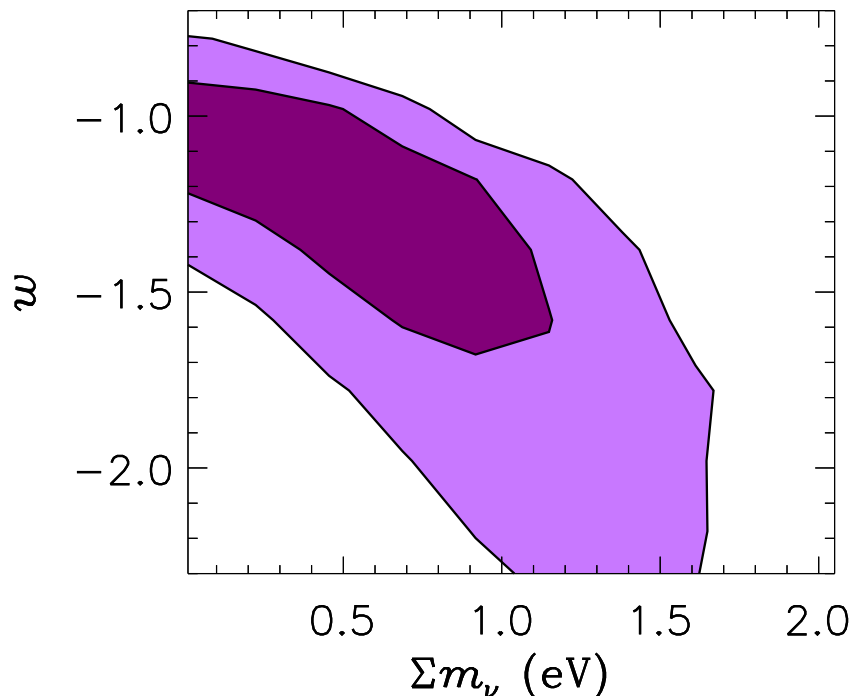


Figure 3: 68% and 95% allowed contours as a function of neutrino mass and dark energy equation of state using WMAP, SDSS, HST, and SNI-a data.

The reason for the degeneracy is that when Ω_ν is increased, Ω_m must be increased correspondingly in order to produce the same power spectrum. However, when $w = -1$ an increasing Ω_m quickly becomes

incompatible with the supernova data. This can be remedied by simultaneously decreasing w because of the well-known Ω_m, w degeneracy in the supernova data. For the particular data set used in this case the 95% C.L. bound changes from 0.65 eV with $w = -1$ to 1.5 eV with free w .

4.2.2 Combining measurements of m_ν and N_ν .

The limits on neutrino masses discussed above apply only for neutrinos within the standard model, i.e. three light neutrinos with degenerate masses (if the sum is close to the upper bound). However, if there are additional neutrino species sharing the mass, or neutrinos have significant chemical potentials this bound is changed. Models with massive neutrinos have suppressed power at small scale, with suppression proportional to Ω_ν/Ω_m . Adding relativistic energy further suppresses power at scales smaller than the horizon at matter-radiation equality. For the *same* matter density such a model would therefore be even more incompatible with data. However, if the matter density is increased together with m_ν , and N_ν , excellent fits to the data can be obtained. This effect is shown in Fig. 4.

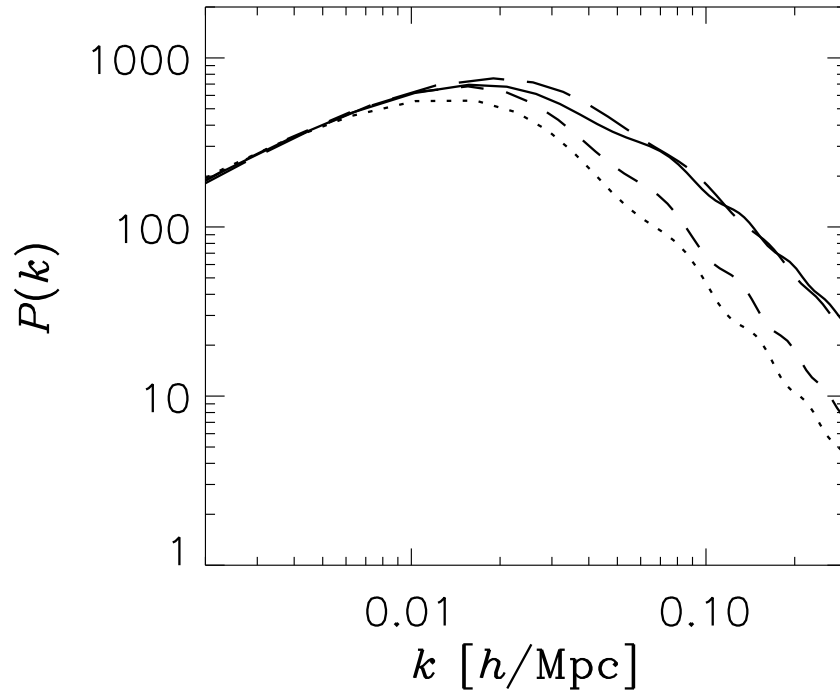


Figure 4: Power spectra for Λ CDM models with $\Omega_b = 0.05$, $\Omega = 1$, $h = 0.7$, $n_s = 1$, and $N_{\nu, \text{massive}} = 1$ and a common large-scale normalization. The full line is for $\Omega_\nu = 0$, $\Omega_m = 0.25$, $N_\nu = 3$, dashed is for $\Omega_\nu = 0.05$, $\Omega_m = 0.25$, $N_\nu = 3$, dotted is for $\Omega_\nu = 0.05$, $\Omega_m = 0.25$, $N_\nu = 8$, and long-dashed is for $\Omega_\nu = 0.05$, $\Omega_m = 0.35$, $N_\nu = 8$ (from [74]).

The effect on likelihood contours for (Ω_ν, N_ν) can be seen in Fig. 5 which is for the case where N_ν species the total mass equally.

A thorough discussion of these models can be found in Refs. [74, 75].

4.2.3 Future neutrino mass measurements

The present bound on the sum of neutrino masses is still much larger than the mass difference, $\Delta m_{23} \sim 0.05$ eV [88, 4], measured by atmospheric neutrino observatories and K2K. This means that if the sum of neutrino masses is anywhere close to saturating the bound then neutrino masses must be almost

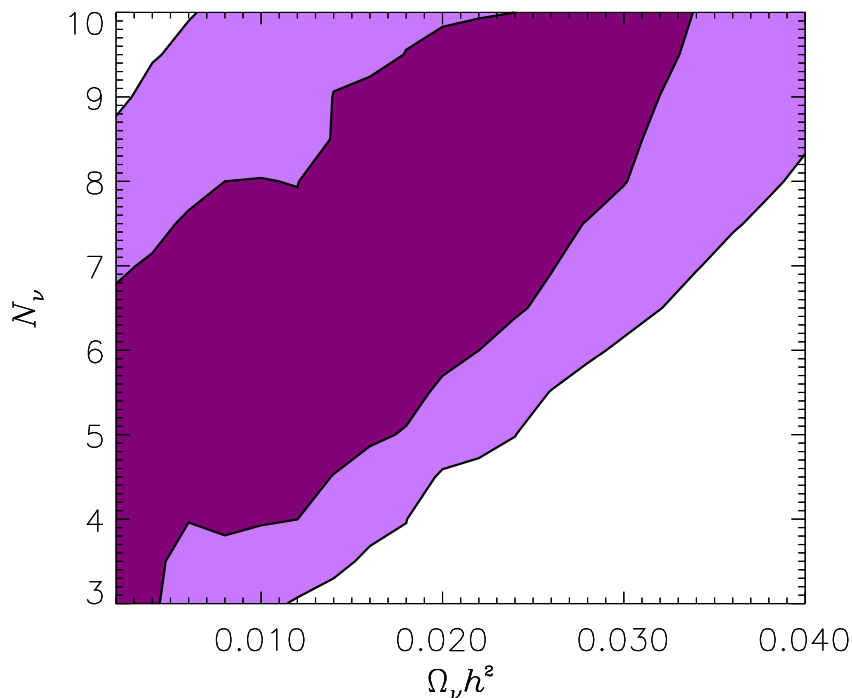


Figure 5: Likelihood contours (68% and 95%) for the case of N_ν neutrinos with equal masses, calculated from WMAP and 2dF data (from [74]).

degenerate. The question is whether in the future it will be possible to measure masses which are of the order Δm_{23} , i.e. whether it can be determined if neutrino masses are hierarchical.

By combining future CMB data from the Planck satellite with a galaxy survey like the SDSS it has been estimated that neutrino masses as low as about 0.1 eV can be detected [83, 84]. Another possibility is to use weak lensing of the CMB as a probe of neutrino mass. In this case it seems likely that a sensitivity below 0.1 eV can also be reached with CMB alone [90]. Possibly the sensitivity could be increased even further with future galaxy cluster surveys [91].

As noted in Ref. [84] the exact value of the sensitivity at this level depends both on whether the hierarchy is normal or inverted, and the exact value of the mass splittings.

4.3 Neutrino warm dark matter

While CDM is defined as consisting of non-interacting particles which have essentially no free-streaming on any astronomically relevant scale, and HDM is defined by consisting of particles which become non-relativistic around matter radiation equality or later, warm dark matter is an intermediate. One of the simplest production mechanisms for warm dark matter is active-sterile neutrino oscillations in the early universe [103, 104, 105, 106, 107].

One possible benefit of warm dark matter is that it does have some free-streaming so that structure formation is suppressed on very small scales. This has been proposed as an explanation for the apparent discrepancy between observations of galaxies and numerical CDM structure formation simulations. In general simulations produce galaxy halos which have very steep inner density profiles $\rho \propto r^\alpha$, where $\alpha \sim 1 - 1.5$, and numerous subhalos [92, 93]. Neither of these characteristics are seen in observations and the explanation for this discrepancy remains an open question. If dark matter is warm instead of cold, with a free-streaming scale comparable to the size of a typical galaxy subhalo then the amount of substructure is suppressed, and possibly the central density profile is also flattened [94, 95, 96, 97, 98, 99, 100]. In

both cases the mass of the dark matter particle should be around 1 keV [108, 109], assuming that it is thermally produced in the early universe.

On the other hand, from measurements of the Lyman- α forest flux power spectrum it has been possible to reconstruct the matter power spectrum on relatively small scales at high redshift. This spectrum does not show any evidence for suppression at sub-galaxy scales and has been used to put a lower bound on the mass of warm dark matter particles of roughly 500 eV [101]. An even more severe problem lies in the fact that star formation occurs relatively late in warm dark matter models because small scale structure is suppressed. This may be in conflict with the low- l CMB temperature-polarization cross correlation measurement by WMAP which indicates very early reionization and therefore also early star formation. One recent investigation of this found warm dark matter to be inconsistent with WMAP for masses as high as 10 keV [94].

The case for warm dark matter therefore seems quite marginal, although at present it is not definitively ruled out by any observations.

5 Discussion

Here I have reviewed some of the basics of neutrino cosmology with particular emphasis on the role that neutrinos play in cosmological structure formation. Exactly because neutrinos play such a crucial role in cosmology it is possible to use cosmological observations to probe fundamental properties of neutrinos which are not easily accessible in lab experiments. Particularly the measurement of absolute neutrino masses from CMB and large scale structure data has received significant attention over the past few years.

Another cornerstone of neutrino cosmology is the measurement of the total energy density in non-electromagnetically interacting particles. For many years Big Bang nucleosynthesis was the only probe of relativistic energy density, but with the advent of precision CMB and LSS data it has been possible to complement the BBN measurement. At present the cosmic neutrino background is seen in both BBN, CMB and LSS data at high significance.

Finally, cosmology can also be used to probe the possibility of neutrino warm dark matter, which could be produced by active-sterile neutrino oscillations.

In the coming years the steady stream of new observational data will continue, and the cosmological bounds on neutrino will improve accordingly. For instance, it has been estimated that with data from the upcoming Planck satellite it could be possible to measure neutrino masses as low as 0.1 eV, and possibly even lower than that if data from future cluster surveys is used.

Neutrino cosmology is certainly an exiting field which is destined to play a key role in neutrino physics in the coming years.

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